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On locally invertible harmonic mappings of plane domains

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Abstract

We prove several global univalence theorems for locally invertible harmonic mappings with certain prescribed boundary behavior in simply and multiply connected domains. In particular, we consider mappings with singular boundary points when the argument principle is not applicable. In addition, we examine harmonic mappings connected with the famous von Mises coordinates. It is shown the univalence of every locally univalent von Mises harmonic mapping defined in a domain convex in the direction of ordinate axis.

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Keywords

argument principle, Hadamard's theorem, Harmonic mapping, local homeomorphism, Mises coordinates

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